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A Fixed Point Theorem In Fuzzy Metric Space Using Weakly Biased Maps

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ABSTRACT:
The main aim of this paper is to prove a common fixed point theorem for weakly biased mappings satisfying the E.A property on a non complete fuzzy metric space.Also this result does not required continuity of the self maps.This theorem Generalizes the result of V.srinivas B.V.B.Reddy and R.Umamaheswarrao [1] in Fuzzy metric spaces.
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I. INTRODUCTION:

Generalizing the concept of commuting maps ,Sessa introduced the concept of weakly commuting maps. Afterwards in 1986, Jungck generalized the concept of weekly commuting mappings by introducing compatible maps. Further ,generalization of compatible maps are given by many others like jungck Murthy and Cho [6].In 1995 ,Jungck and Pathak [7] introduced the concept of biased maps in the same paper they also given the weakly biased mappings which is generalization of biased maps..Property E.A introduced by M.Aamri and D.EI Moutawakil .

II. PRELIMINARIES:

Definition 1:.Let A and B two self of a maps fuzzy metri space (X,M,*) in itself .The maps A and S are said to be compatible if for all t>0 $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z\in X$

Definition 2.:
Let A and S maps from a fuzzy metric space (X,M,*) into itself ,The maps are said to be weakly compatible if they commute at their co incidence points that is,Az=Sz implies that ASz=SAz

Definition 3:.let A and B be two self maps of fuzzy metric space (X,M,*).we say that A and S are satisfy the proparty (E.A) if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z\in X$.

Lemma 4: for all x,y $\in$X ,t>0 and for a number k$\in$(0,1) then x=y.

Definition 5: A and S are weakly S- biased in fuzzy metric space if Ap=Sp implies M(SA(p),S(p),t)≥M(AS(p),A(p),t)for some p$\in$X

Definition 6 A and S are weakly A- biased in fuzzy metrc space if Ap=Sp implies M(AS(p),A(p),t)≥M(SA(p),S(p),t) for some p$\in$X

Remark 7.: Weakly compatible maps are Weakly biased but converse not trueThe maps satisfying the property E.A are not taken to be continuous at the fixed point.

The completeness of the space is replaced by the closeness of one of the range of maps

Proposition 8: let {A,S} ,be a pair of elf maps of a fuzzy metric space (X,M,*).If the pair {A,S} is

weakly compatible then it is either weakly A- biased or weakly S- biased.

III. Main result.

The following lemma playing an important role in main theorem**.**

Lemma :Let A,B, S and T be self maps of metric space (X,M,*) satisfying the following Conditions.

$(i)\, A(X)\subseteq T(X)\;\; B(X)\subseteq S(X)$

$(ii)\left[M\,(Ax,By,Kt)\right]^2 * M\,(Ax,By,Kt)$

$$M\,(Ty,Sx,Kt)\geq\left\{\begin{matrix} k_1\left[M\,(By,Sx,1.25kt)\right]* \\ \left[M\,(Ax,Ty,1.25kt)\right]+ \\ k_2\left[M\,(Ax,Sx,2.5Kt)\right] \\ *\left[M\,(By,Ty,2.5Kt)\right]\end{matrix}\right\}M\,(Ty,Sx,Kt)$$

for x,y *in* X *and* $k_1,k_2\geq 0,\; k_1+k_2\geq 1$

(iii) *One of the* $A(x),T(x),B(x)$ *and* $S(x)$ *is closed.*

(iv) *One of pairs* (A,S) *and* (B,T) *satisfies the properity E.A*

Then each of the pairs (A,S) and (B,T) has a coincident point.

Proof.

Suppose that the pairs $(\mathrm{B,T})$ satisfies E.A property then

there exists a Squence $\{x_n\}$ *in X Suchthat*

$$\underset{n\to\infty}{Lt} Bx_n=\underset{n\to\infty}{Lt} Tx_n=z \;\; \text{for some } z\in X$$

Since $B(x)\subseteq T(x)$ *there exsist a Squence* $\{y_n\}$ *in X Suchthat*

$Bx_n=Sy_n$ so $\underset{n\to\infty}{Lt} Sy_n=z$

We now show that $\underset{n\to\infty}{Lt} Ay_n=z$

put $x=y_n\;\; y=x_n$ *in* (ii)

$$\left[M\,(Ay_n,Bx_n,kt)\right]^2 * M\,(Ay_n,Bx_n,kt)\,M\,(Tx_n,Sy_n,kt)\geq$$

$$\left\{\begin{matrix} k_1\left[M\,(Bx_n,Sy_n,1.25kt)\right]* \\ \left[M\,(Ay_n,Tx_n,1.25kt)\right]+ \\ k_2\left[M\,(Ay_n,Sx_n,2.5Kt)\right]* \\ \left[M\,(Bx_n,Tx_n,2.5Kt)\right]\end{matrix}\right\}M\,(Tx_n,Sy_n,Kt)$$

$$\left[M\,(Ay_n,z,kt)\right]^2 * M\,(Ay_n,z,kt)\,M\,(z,z,kt)$$

$$\geq\left\{\begin{matrix} k_1\left[M\,(z,z,1.25kt)\right]* \\ \left[M\,(Ay_n,z,1.25kt)\right]+ \\ k_2\left[M\,(Ay_n,z,2.5Kt)\right]* \\ \left[M\,(z,z,2.5Kt)\right]\end{matrix}\right\}M\,(z,z,Kt)$$

$$\left[M\,(Ay_n,z,kt)\right]\begin{bmatrix} M\,(Ay_n,z,kt)* \\ M\,(z,z,kt)\end{bmatrix}\geq\left\{\begin{matrix} k_1\left[M\,(Ay_n,z,1.25kt)\right]+ \\ k_2\left[M\,(Ay_n,z,2.5Kt)\right]\end{matrix}\right\}$$

$$M\,(Ay_n,z,kt)\geq k_1+k_2$$
$$M\,(Ay_n,z,kt)\geq 1$$
$$Ay_n=z$$

Suppose that S(X) is closed subspace of X then z=Su for some u∈X

Now replace x=u,y=x_{2n+1} in (ii) we have

$$\left[M(Au,Bx_{2n+1},Kt)\right]^2 * M(Au,Bx_{2n+1},Kt)\,M(Tx_{2n+1},Su,Kt) \geq \left\{\begin{array}{l} k_1\left[M(Bx_{2n+1},Su,1.25kt)\right]* \\ \left[M(Au,Tx_{2n+1},1.25kt)\right]+ \\ k_2\left[M(Au,Su,2.5Kt)\right]* \\ \left[M(Bx_{2n+1},Tx_{2n+1},2.5Kt)\right] \end{array}\right\} M(Tx_{2n+1},Su,Kt)$$

$$\left[M(Au,z,kt)\right]^2 * \begin{bmatrix} M(Au,z,kt) \\ M(z,z,kt) \end{bmatrix} \geq \left\{\begin{array}{l} k_1\left[M(z,z,1.25kt)\right]* \\ \left[M(Au,z,1.25kt)\right]+ \\ k_2\left[M(Au,z,2.5kt)\right]* \\ \left[M(z,z,2.5Kt)\right] \end{array}\right\}(z,z,kt)$$

$$\left[M(Au,z,Kt)\right]^2 \geq \left\{\begin{array}{l} k_1\left[M(Au,z,1.25kt)\right]+ \\ k_2\left[M(Au,z,2.5Kt)\right] \end{array}\right\}$$

$$\left[M(Au,z,Kt)\right] \geq \left\{k_1 + k_2\right\}$$

$$\left[M(Au,z,Kt)\right] \geq 1$$

Hence Au=z=Su consequently u is coincidence point of the pair (A,S)

From A(X)⊆T(X) which gives z∈T(X) we declare that there exists v∈X such that Tv=z

To prove Bv=z suppose that Bv≠z

We have x=u,y=v

$$\left[M(Au,Bv,kt)\right]^2 * \begin{bmatrix} M(Au,Bv,kt) \\ M(Tv,Su,kt) \end{bmatrix} \geq \left\{\begin{array}{l} k_1\left[M(Bv,Su,1.25kt)\right]* \\ \left[M(Au,Tv,1.25kt)\right]+ \\ k_2\left[M(Au,Su,2.5kt)\right]* \\ \left[M(Bv,Tv,2.5kt)\right] \end{array}\right\} M(Tv,Su,kt)$$

$$\left[M(z,Bv,kt)\right]^2 * \begin{bmatrix} M(z,Bv,kt) \\ M(z,z,kt) \end{bmatrix} \geq \left\{\begin{array}{l} k_1\left[M(Bv,z,1.25kt)\right]* \\ \left[M(z,z,1.25kt)\right]+ \\ k_2\left[M(z,z,2.5kt)\right]* \\ \left[M(Bv,z,2.5kt)\right] \end{array}\right\} M(z,z,kt)$$

$$\left[M(z,Bv,kt)\right]^2 \geq \left\{\begin{array}{l} k_1\left[M(Bv,z,1.252kt)\right]+ \\ k_2\left[M(Bv,z,2.5Kt)\right] \end{array}\right\}$$

$$\left[M(z,Bv,kt)\right] \geq \left\{k_1 + k_2\right\}$$

$$\left[M(z,Bv,kt)\right] \geq 1$$

$$Bv = z$$

Therefore (B,T) has a coincident point v.

The same result holds if we suppose that one of A(X), B(X),and T(X) is closed subspace of X

Theorem . the pairs (A,S) and (B,T) are weakly S biased and weakly T biased respectively then A,B,S and T have a unique common fixed point .

Proof:

S(X) is closed subspace of X and and z,u,v be as in lemma.

By lemma u,v are coincident points of (A,S) and (B,T) respectively Since the pair (A,S) is weakly s biased then Au=Su implies M(SAu,Su,t)≥M(ASu,Au,t) which gives M(Sz,z,t)≥M(Az,z,t)

Bv=Tv and T weakly biased compatible M(TBv,Tv,t)≥M(BTv,Bv,t) gives M(Tz,z,t)≥M(Bz,z,t)

Put x=z,y=v in (ii)

$$\left[M(Az,Bv,kt)\right]^2 * \begin{bmatrix} M(Az,Bv,kt) \\ M(Tv,Sz,kt) \end{bmatrix} \ge \left\{ \begin{array}{l} k_1\left[M(Bv,Sz,1.25kt)\right]* \\ \left[M(Az,Tv,1.25kt)\right] + \\ k_2\left[M(Az,Sz,2.5kt)\right]* \\ \left[M(Bv,Tv,2.5kt)\right] \end{array} \right\} M(Tv,Sz,kt)$$

$$\left[M(Az,z,kt)\right]^2 * \begin{bmatrix} M(Az,z,kt) \\ M(z,Sz,kt) \end{bmatrix} \ge \left\{ \begin{array}{l} k_1\left[M(z,Sz,1.25kt)\right]* \\ \left[M(Az,z,1.25kt)\right] + \\ k_2\left[M(Az,Sz,2.5kt)\right]* \\ \left[M(z,z,2.5kt)\right] \end{array} \right\} M(z,Sz,kt)$$

$$\left[M(Az,z,kt)\right]^2 * \begin{bmatrix} M(Az,z,kt) \\ M(z,Sz,kt) \end{bmatrix} \ge \left\{ \begin{array}{l} k_1\left[M(z,Sz,1.25kt)\right]* \\ \left[M(Az,z,1.25kt)\right] + \\ k_2\left[M(Az,z,2.5kt)\right]* \\ \left[M(z,Sz,2.5kt)\right] \end{array} \right\} M(z,Sz,kt)$$

$$\left[M(Az,z,kt)\right]^2 \ge \left\{ \begin{array}{l} k_1\left[M(Az,z,1.25kt)\right] + \\ k_2\left[M(Az,z,1.25kt)\right]*\left[M(z,Az,kt)\right] \end{array} \right\} M(z,Az,kt)$$

$$\left[M(Az,z,kt)\right] \ge \left\{k_1 + k_2\right\}\left[M(Az,z,1.25kt)\right]$$

$$\left[M(Az,z,kt)\right] \ge \left[M(Az,z,1.25kt)\right]$$

$$Az = z$$

Put x=u,y=z in(ii)

$$\left[M(Au,Bz,kt)\right]^2 * M(Au,Bz,kt)\, M(Tz,Su,kt) \ge \left\{ \begin{array}{l} k_1\left[M(Bz,Su,1.25kt)\right]* \\ \left[M(Au,Tz,1.25kt)\right] + \\ k_2\left[M(Au,Su,2.5kt)\right]* \\ \left[M(Bz,Tz,2.5kt)\right] \end{array} \right\} M(Tz,Su,kt)$$

$$\left[M(z,Bz,kt)\right]^2 * M(z,Bz,kt)\, M(Tz,z,kt) \ge \left\{ \begin{array}{l} k_1\left[M(Bz,z,1.25kt)\right]* \\ \left[M(z,Tz,1.25kt)\right] + \\ k_2\left[M(z,z,2.5kt)\right]* \\ \left[M(Bz,Tz,2.5kt)\right] \end{array} \right\} M(Tz,z,kt)$$

$$\left[M(z,Bz,kt)\right]^2 * M(z,Bz,kt)\, M(Tz,z,kt) \ge \left\{ \begin{array}{l} k_1\left[M(Bz,z,1.25kt)\right]* \\ \left[M(z,Tz,1.25kt)\right] + \\ k_2\left[M(Bz,Tz,2.5kt)\right] \end{array} \right\} (Tz,z,kt)$$

$$\left[M(z,Bz,kt)\right]^2 * M(z,Bz,kt)\, M(Tz,z,kt) \ge \left\{ \begin{array}{l} k_1\left[M(Bz,z,1.25kt)\right]* \\ \left[M(z,Tz,1.25kt)\right] + \\ k_2\left[M(Bz,z,2.5kt)\right]* \\ \left[M(z,Tz,2.5kt)\right] \end{array} \right\} M(Tz,z,kt)$$

$$\left[M(z,Bz,kt)\right] \ge \left\{k_1\left[M(Bz,z,1.25kt)\right] + k_2\left[M(Bz,z,1.25kt)\right]\right\}$$

$$\left[M(z,Bz,kt)\right] \ge \left\{k_1 + k_2\right\}\left[M(Bz,z,1.25kt)\right]$$

$$\left[M(z,Bz,kt)\right] \ge \left[M(Bz,z,1.25kt)\right]$$

$$Bz = z$$

We have M(Sz,z,t)≥M(Az,z,t) gives
M(Sz,z,t)≥M(z,z,t) impliea Sz=z.
Therefore Sz=Az=z

The uniqueness of common fixed point can easly proved

Example: Let X=[0,10) with the metric d(x,y)=|x-y| and for some t>0 define

$$M(x,y,t)=\begin{cases}\dfrac{t}{t+|x-y|} & if\ \ t>0\\ 0 & if\ \ t=0\end{cases} \quad \text{X}$$

$for\ \ x,y\in X$

We define mappings A,B,S and T on X by

$$A(x)=\begin{cases}\dfrac{x}{2} & if\ \ 0\le x<10\\ 1 & if\ \ x=1\\ 0.9 & if\ 1<x<0\end{cases}$$

$$B(x)=\begin{cases}\dfrac{3}{4} & if\ \ 0\le x<10\\ 1 & if\ \ x=1\\ 1 & if\ 1<x<0\end{cases}$$

$$S(x)=\begin{cases}\dfrac{1}{4} & if\ \ 0\le x<10\\ \dfrac{1}{x} & if\ \ x=1\\ \dfrac{1}{x^2} & if\ 1<x<0\end{cases}$$

$$T(x)=\begin{cases}\dfrac{3x}{2} & if\ \ 0\le x<10\\ x^2 & if\ \ x=1\\ x^2 & if\ 1<x<0\end{cases}$$

We observe that A(X)⊆T(X) and B(X)⊆S(X), B(X) is closed subspace of X.The pair (B,T) satisfying property E.A with the sequence x_n=1+(1/n) n≥1 .The coincident points of the pairs (A,S) and (B,T) are 1 and ½.The pairs (A,S) is A-weakly biased and The pair (B,T) is B-weakly biased at the coincident points .but which is not compatible and not weakly compatible.

IV.REFERENCES:

[1] V Srinivas, B.V.B.Reddy, R.Umamaheswarrao, *A Common fixed point Theorem on Fuzzy metric Spaces*, Kathmandu University journal of Science ,Engineering and Technology,8 (II)(2012)77-82.

[2] P.P.Murthy, M.R.Singh, L.S.Singh, *weakly Biased maps as a generalization of occasionally weakly compatible maps*, International Journal of Pure and Applied Mathematics ,91(2) 2014,143-153.

[3] M.R Singh ,Y Mahendra Singh,*Fixed points for biased maps on metric space*, Int.J.Contemp.Math.Sci.Vol.4(2009),no.16, 769-778.

[4] H.K.Pathak, R.R Lopez and R.K Verma, *A Common fixed point theorem using implicit relation and Property (E.A) in metric Spaces*, Filomat 21(2)(2007),211-234.

[5] S.Sharma, *Common fixed point theorems in fuzzy metric space*, Fuzzy sets and Systems,127(2002),345-352.

[6] G.Jungck P.P Murthy, Y.J.Cho, *Compatible mappings of type(A) and Common fixed points*, Math, Japon, 38 (2)(1993) 381-390.

[7] G.jungck, H.H Pathak, Fixed points via biased maps, Proc.Amer.Math.Soc.123(7)(1995) 2049-2060.

[8] MAamri and D.EI Motawakil, *Some common fixed point theorems under strict contractive conditions*, Journal of mathematical Analysis and applications,J.Math.Annal.Appal.270(2002 0 181-188